Size independence of the strength of snow

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The mechanical reliability of 600 randomly taken snow samples follows Weibull distributions: If σ_{max} is the maximum stress present in a specimen of given density, the fraction of specimens that fail at stresses below σ_{max} is $P=1-\exp[-(\sigma_{max}/\sigma_0)^m]$. The scale parameter σ_0 evaluated by the maximum likelihood method increases nearly quadratically with the density ρ of snow, but, unlike predicted by the weakest link model, is independent of size and shape of the specimen: there is no size dependence of the strength of snow. The Weibull parameter m is independent of density, size, and shape of the snow sample, $m=1.5\pm0.5$. This implies, on the one hand, that the results of laboratory scale tests can be used for avalanche prediction, but on the other hand, that these predictions remain contaminated with large statistical errors. Snow is a fragile, weak, and unreliable material.

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I. INTRODUCTION

Snow is a foam of ice [1,2]. Given the fact it is usually observed within 10% of the melting point, solid ice (density 917 kg/m^3) is a remarkably strong, tough, and reliable material. The yield stress is about 10 MPa [3,4], the fracture toughness K_{Ic} is about 115–250 kPa m$^{1/2}$ [5,6], and the Weibull exponent is about $m=4\pm1$ for freshwater ice [7]. The same cannot be said about snow, which is a remarkably weak, brittle, and unreliable material. Snow of 18% density of the density of ice has a yield strength of about 100 kPa [1,8,9], a fracture toughness of about $K_{Ic}=400$ Pa m$^{1/2}$ in tension, and $K_{IIc}=400$ Pa m$^{1/2}$ in shear [10–12]. In preliminary experiments a Weibull modulus of $m=2.1$ has been reported [13]. The disappointing properties of snow can be understood from the foam theory [14,15], which predicts spectacular deterioration of all mechanical properties of foams with decreasing density. In practice, the mechanical properties of snow are relevant for avalanche theories and risk assessment. Given that about 100 people die annually in USA, and about the same in Europe in slab avalanches, the matter is worth pursuing. The shear strength is relevant for mechanical models of slabs that break under its own weight [16], the toughness is the decisive parameter for slab avalanches triggered by skiers [12], and the low Weibull modulus for the haphazardness of avalanches in general. The reliability is relevant insofar as, even if nominally the snow cover should hold under a given stress, the statistics might lead to unforeseeable avalanche risks that can be, and often are, mortal. In this paper we concentrate on the reliability aspect of snow. So far the Weibull modulus $m=2.1$ has only been measured once for snow of 140 kg/m^3 density [13]. This is an extraordinarily low value. The higher the value of m, the more deterministic is the failure; typical values are $m=22$ for reactor steel [17], $m=3$–10 for freshwater, iceberg, and seawater ice [7], about 12 for coal and rock [18], and $m=10$–25 for ceramics [19]. Recently $m=8$ has been measured for an aluminum foam [20]. These measurements, so far the only ones on foams, indicate that m for a foam is less than for the solid it is made of. In other words, foams are not only weaker, but also less reliable than their solids. It should be recalled that the lower the value of m, the higher is the probability that a specimen breaks under, say 10% of the nominal stress it is supposed to sustain. For snow, as a foam of ice, not only lower fracture strain, but also less reliability than ice is expected. We will prove this to be the case; and, for the worse, the statistical theory predicts that the lower the value of m, the stronger is the size dependence of strength. The physical explanation is that weakness is associated with defects in the material, and the larger the material volume under examination, the greater is the probability to have fatal defects present. In the avalanche context the predicted size dependence is perturbing, the strength is predicted to vary like the power of $-1/m$ of the specimen volume. With approximately $m=2$, this forebodes ill for the strength of snow slabs in nature (tens to hundreds of cubic meters) with the (already low) values of yield stress measured in the laboratory size specimens (a few cubic decimeters). In this paper we will show that there is no size dependence, and extrapolation from laboratory to field size slabs is justified. Finally, in fracture mechanics simulations of snow avalanches [21] the statistical distribution of strengths plays a determining role. First, the median and the width of the strength distribution must be known, second, it must be known if the assumed distribution depends itself on the size of the specimen or not. Also in analytic fracture mechanics the question if, for example, the shear mode fracture toughness depends on the snow slab thickness arises [22].

II. STATISTICAL THEORY OF FAILURE

The statistical theory of random strength, proposed by Weibull [23–25], is the mathematical formulation of two hypotheses.

(1) The specimen fails as soon as one small volume ele-

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ment, wherever it is, attains the strength limit. This is commonly referred to as the weakest link model.

(2) The strength limit is a random variable. It is assumed that the strength of the entire specimen is limited by the strength of the weakest link of small constituent pieces.

The two assumptions arise naturally from three physical conditions: (a) defects control the strength, but do not interact, (b) there is a relation between defect size and strength (usually Griffith's law), and (c) the number of defects follows a power law in defect size. These assumptions are more or less justified for various materials; the proof is in the pudding: if the materials follow the (Weibull) statistics that results, the hypothesis is justified. If at stress σ the probability of failure in a small volume is $F(\sigma)$, then the probability distribution of the smallest strength, in a random sample composed of such volumes, is

$$P(\sigma)=1-\exp[-nF(\sigma)]. \tag{1}$$

For large n the function $(1-F)^n$ can be approximated by $\exp(-nF)$, and the probability of survival becomes

$$P(\sigma)=1-[1-F(\sigma)]^n. \tag{2}$$

With Weibull we assume as constitutive law of failure a power law without threshold stress,

$$F(\sigma)=(\sigma/\Sigma_0)^m. \tag{3}$$

The parameter Σ_0 is a property of the material only, not of the specimen or loading geometry. Also m, the Weibull exponent, is a material property. With Eq. (3), Eq. (2) becomes for any volume, which is n times a reference volume V_0, $n=V/V_0$,

$$1-P(\sigma)=\exp\left[-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right]. \tag{4}$$

From Eq. (4) the volume dependence of the stresses for equal probabilities of fracture is directly visible:

$$\frac{\sigma_1}{\sigma_2}=\left(\frac{V_2}{V_1}\right)^{1/m}. \tag{5}$$

If the stress is not constant throughout the specimen, but a function of position [26],

$$\sigma(x,y,z)=\sigma_{max}\,g(x,y,z), \tag{6}$$

the probability of survival is

$$1-P(\sigma)=\exp\left[-\frac{V_E}{V_0}\left(\frac{\sigma_{max}}{\Sigma_0}\right)^m\right] \tag{7}$$

with an effective volume V_E being defined as

$$V_E=\int_V dV\,g(x,y,z)^m=kV. \tag{8}$$

The proportionality constant k between the effective volume V_E and the real volume V depends on specimen volume,

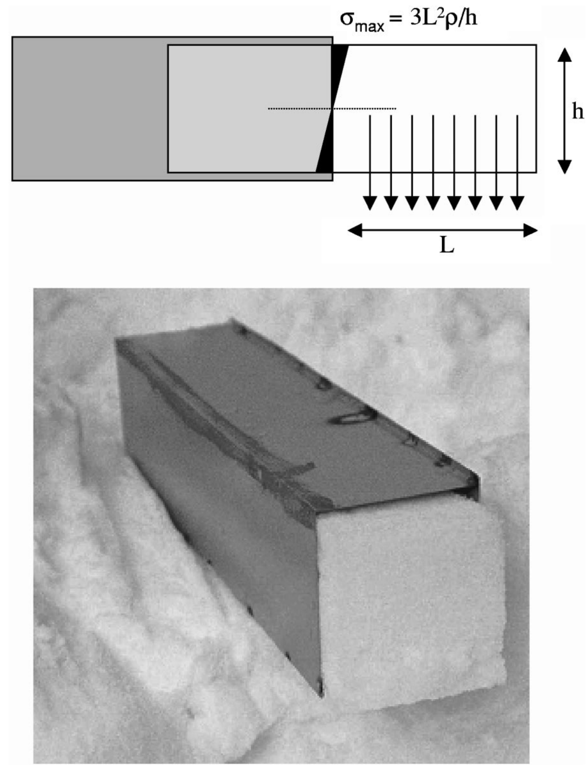

FIG. 1. Cantilever beam geometry used.

shape, and loading, and σ_{max} being the maximum stress within the specimen volume V.

So far a constant specimen volume V, in which the stress $\sigma(x,y,z)$ stems from external loading, was assumed. This is the typical case of ceramic specimens loaded in three-point-bending arrangement. Only under such conditions the maximum stress appears with the power m, the Weibull exponent in the fracture probability. If the body loading is present, for example, beams bending under their own weight, as our specimens, the maximum stress σ_{max} in the volume V of the sample is itself a function of specimen volume. For the specimen used by us, beams of cantilever length L, width t, and height h, and thus of volume Lth as shown in Fig. 1, the maximum stress occurs in the plane of support [27],

$$\sigma_{max}=3L^2\rho/h=3V^2\rho/(t^2h^3). \tag{9}$$

According to beam theory the integral equation (9) is

$$V_E=V/[(2m+1)(m+1)]. \tag{10}$$

For such situations it is advantageous to write the fundamental equation (7) in the form

$$\ln[1-P(\sigma)]=-(\sigma_{max}/\sigma_0)^{m+1/2}, \tag{11}$$

with the scale parameter

$$\sigma_0=[V_0\Sigma_0^m 3^{1/2}\rho^{1/2}(2m+1)(m+1)/(th^{3/2})]^{1/(m+1/2)}. \tag{12}$$

It is important to notice that for such body loading the failure probability is not proportional to the Weibull parameter of the material, m, any more, but to $m+1/2$! Moreover, the simple geometry chosen allows us to examine if the scale parameter σ_0 does indeed scale with the density ρ and the geometric parameters t and h as predicted by Eq. (12) or not.

The most frequently used methods to evaluate Eq. (7) are the linear regression and the maximum likelihood method: we evaluated our data by both the methods, but the results do not differ considerably. In the following we restrict to the method of maximum likelihood for the following reasons.

(1) For maximum likelihood, the fracture probabilities are not arbitrarily chosen, but the fracture probability densities are maximized for a certain set of measured fracture strengths with respect to the Weibull parameters.

(2) The standard deviations are available even in dependence on the number of tests, obtained by either analytical methods or computer simulations [28].

(3) The maximum likelihood method has the lower standard deviation and is thus mathematically more reliable [29]. In the appropriate formulas for external loading [28], we must replace m by $m+1/2$ for our case of body loading. If σ_j is the jth measurement of M measured values of σ_{max}, the Weibull modulus m_{ML} is obtained by the root of the following equation, the index ML denoting the evaluation by maximum likelihood:

$$\frac{M}{m_{ML}+1/2}+\sum_{j=1}^{M}\ln\sigma_j-M\frac{\sum_{j=1}^{M}(\sigma_j)^{m_{ML}+1/2}\ln\sigma_j}{\sum_{j=1}^{M}(\sigma_j)^{m_{ML}+1/2}}=0 \tag{13}$$

and the scale parameter σ_{ML} by

$$M\sigma_{ML}^{m_{ML}+1/2}=\sum_{j=1}^{M}(\sigma_j)^{m_{ML}+1/2}. \tag{14}$$

Because the Weibull distribution is not symmetric, the (usually unknown) true value σ_0 of the distribution is related to σ_{ML} by

$$\sigma_{ML}=\sigma_0 M^{-1/m}\frac{\Gamma\left(M+\frac{1}{m+1/2}\right)}{\Gamma(M)} \tag{15}$$

and the true value of m to m_{ML} by

$$m_{ML}+1/2=(m+1/2)(1+2.1\,M^{-1.1}). \tag{16}$$

The respective standard deviations are

$$(\Delta\sigma_{ML})^2=\sigma_0^2\,1.05\,M^{-2/(m+1/2)}\left\{\frac{\Gamma\left(M+\frac{2}{m+1/2}\right)}{\Gamma(M)}-\left[\frac{\Gamma\left(M+\frac{1}{m+1/2}\right)}{\Gamma(M)}\right]^2\right\} \tag{17}$$

and

$$(\Delta m_{ML}+1/2)^2=(m+1/2)(0.042+2.34M^{-0.88}). \tag{18}$$

The true value σ_0 can only be obtained in the limit $M\to\infty$ or $m\to\infty$. But, as the number of tested specimens M is large in our case and $\sigma_{ML}\cong\sigma_0$ and $m_{ML}\cong m$, we use, for the sake of simplicity, here the following notation:

$$\sigma_{ML}=\sigma_0,\quad m_{ML}=m,\quad \text{justified for large } M. \tag{19}$$

Equation (7) has several consequences.

(1) Only the scale parameter in the combination $\Sigma_0(V_0/V_E)^{1/m}$ appears; neither the effective volume V_E nor the reference stress Σ_0 can be measured separately.

(2) Neither specimen shape nor size appears explicitly in the final expression, shape and size enter only through the integral, Eq. (8), into the effective volume. Two specimens of different shape and size under different loadings are predicted to have the same failure probability as long as their effective volumes V_E according to Eq. (8) and the maximum stress within them, σ_{max}, are the same.

(3) If true, the predictive power of Eq. (7) would be large, because data obtained for one set of specimen geometry could be transferred to other sets of specimen geometries. With a value of m measured once for one set, the effective volume V_E can be calculated for any specimen size and shape and loading geometry, and failure probabilities can be predicted.

(4) For different specimen volumes, the same probability of failure is predicted to occur for the same value of $V_E\sigma_{max}^m$, irrespective of the loading system or specimen geometry. This is a very strong statement and the most critical test for assessing if the weakest link model is applicable. In other words, the weakest link hypothesis implies that the scale parameter σ_0 should depend on shape, size, and loading mode of the specimen. For given specimen geometry and size, V_E is constant, and the Weibull distribution, Eq. (7), merely predicts a sigmoidal variation of $1-P$ with σ_{max} and a bell shaped curve for $dP/d\sigma_{max}$ as a function of σ_{max}. The position of the bell shaped curve is fitted to a stress and the width to a value of m. Another fit would be a normal distribution, which also has two parameters. Such a normal distribution, however, has no interpretation in terms of weakest links. It is clear that fitting an experimental distribution to the Weibull one is not a very stringent test of the weakest link hypothesis. A more stringent test is that if one and the same Weibull distribution fits two different specimen shapes with the same V_E, which, according to the integral, Eq. (8), can be obtained from different loadings. The strictest test, however, is the verification of the volume and shape dependence. The physical interpretation is clear: weakness is associated with the presence of defects, and the larger the specimen volume, the greater is the likelihood of defects being present. According to Weibull, larger specimens must be weaker than small ones, and the size variation is a proportionality of stress with the power $1/m$ of the specimen volume. Experimentally it is not easy to produce specimens of different sizes to the same quality specifications and to test them under identical conditions (for example, the stiffness of the

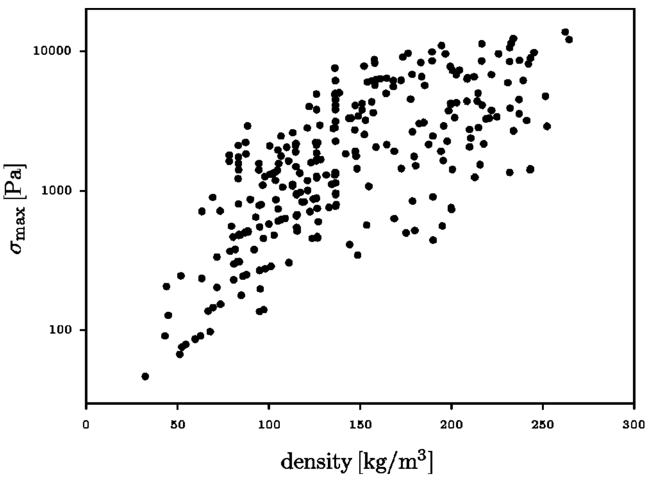

FIG. 2. Perla's [31] strength data, obtained on 276 cantilever beams of $h = 5$ cm height, and $t = 30$ cm width.

testing machine must remain constant relative to the specimen, deformation rates must be the same, etc.). It is not surprising, therefore, that more often than not the $1/m$ power law is not confirmed, even though a good fit with m is possible at constant volume [7,30].

III. EXPERIMENTAL SETUP

In order to be of any applicability for field work, a simple specimen shape must be chosen. Following Perla [31], we used cantilever beams of rectangular or quadratic cross section, which break under their own weight. The geometry is shown in Fig. 1. In practice, the beam is pushed forward until it breaks, the length L of the cantilever at fracture and the density of the broken off piece are measured, and the maximum stress σ_{max} is calculated according to Eq. (9). The Weibull parameters can then be evaluated either by the appropriate method, e.g., linear regression or maximum likelihood.

IV. DENSITY DEPENDENCE OF RELIABILITY

Perla [31] let 276 cantilever beam specimens of various density break under its own weight. He did not specify the temperature at which the mechanical tests were conducted, but it must have been a few degrees below freezing. He chose a height of $h = 5$ cm and a width of $t = 30$ cm. Thirty-five years ago snow was still considered as a material somehow peculiar, and not yet identified as foam of ice. Perla did not identify stresses, failure probability, and the like, but expressed his results in terms of "beam numbers." From his data it is easy to determine the maximum stress present in his samples, and Fig. 2 shows the strength values σ_{max} obtained as a function of snow density. At that time the fact that the strength of snow increased with density was already considered as an important result on its own, and no statistical analysis was attempted. Today a more sophisticated analysis is possible. Even today, however, no theory that would allow us to consider Weibull statistics as a function of one continuous parameter (in our case the density) is available. Therefore, we grouped the data of Fig. 2 according to density into $276/M$ batches of M members each, and applied the maximum likelihood analysis to the groups. The choice of batch size is not obvious: in each batch there is a variation of density, which increases with batch size, but on the other hand the errors Eq. (17) and Eq. (18) decrease with batch size [28,29]. As example of our data, Fig. 3 shows the probability of failure for division of the 276 measurements into three groups of densities: $32 < \rho < 115$, $115 < \rho < 168$, and $168 < \rho < 265$ kg/m^3. First, we determined m from Eq. (13) and σ_0 from Eq. (14) and the errors $\Delta\sigma_0$ and Δm from Eq. (17) and Eq. (18), respectively. Figure 4 shows the variation of the shape parameter σ_0 and of the Weibull exponent m with density, Fig. 4(a) for the three batches of 92 samples of Fig. 3, Fig. 4(b) for nine batches of 30 samples (one batch 36 samples) each, and Fig. 4(c) for 23 batches of 12 members each. Apparently, the scale parameter σ_0 is rather insensitive to the method of analysis and increases nearly quadratically with density (with an exponent in a power law between 1.8 and 1.9 for the respective batches). The increase must be due to the known genuine increase of strength with density [8,9]. It does not vary with the power $\rho^{1/(2m+1)} = 1/4$ as predicted by Eq. (12). The Weibull modulus, on the other hand, is associated with a large statistical error. It is independent of the batch size choice and of the density, extraordinarily low, about $m = 1.5 \pm 0.5$. All densities of snow, between 30

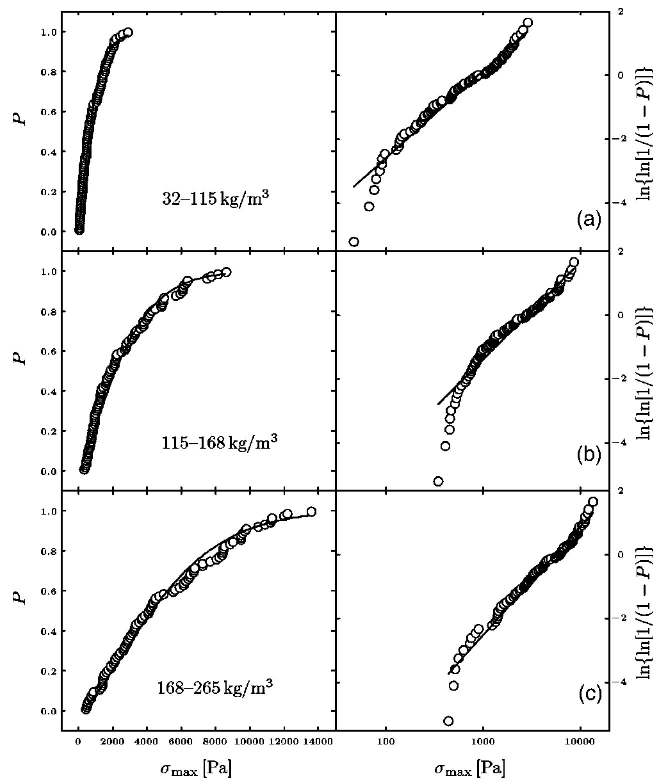

FIG. 3. Probability of failure as a function of the maximum stress σ_{max} in Perla's samples, both in linear and logarithmic scale. (a) Densities between 32 and 115 kg/m^3, (b) densities between 115 and 168 kg/m^3, and (c) densities between 168 and 250 kg/m^3.

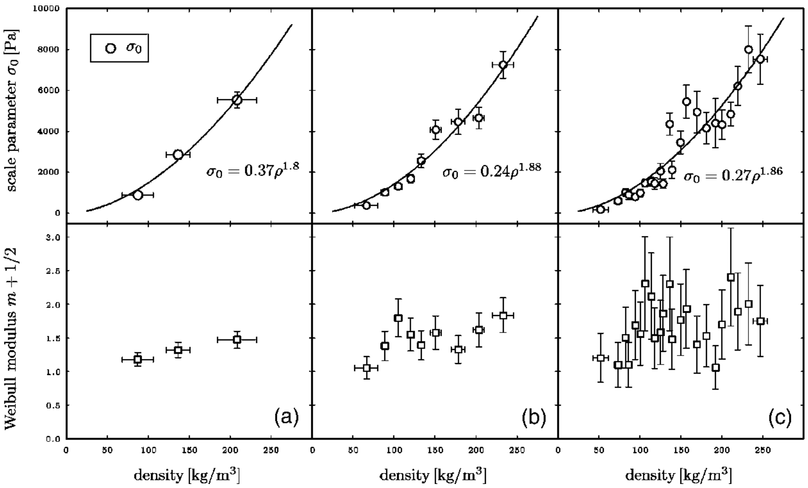

FIG. 4. Weibull analysis of Perla's [31] data. The scale parameter σ_0 and the Weibull modulus m as a function of density. For the 270 specimens (a) divided into three density groups, (b) divided into nine density groups, and (c) divided into 23 density groups. Values and error bars according to Eqs. (13)–(18).

and 260 kg/m^3, are equally unreliable and their mechanical behavior unpredictable.

V. SHAPE DEPENDENCE OF RELIABILITY

We tested 180 cantilever beams of snow of density 107 ± 10 kg/m^3 of widths $t=10$ cm, 60 of height 3 cm, 5 cm, and 10 cm, respectively. Tests were conducted at -5 ± 2 °C. The mean cantilever lengths at which they broke under their own weight were 9, 13.5, and 17.6 cm, which amounts to three sets of volumes in the proportion 1:2.5:6.7. From Eq. (9) the maximum stress σ_{max} in the cantilevers (at the upper surface, above the support) was calculated. Figure 5 shows the three Weibull sets of data in linear and logarithmic coordinates, respectively. Weibull analysis gives the scale parameters as 1120 ± 70, 1221 ± 57, and 1156 ± 71 Pa, and the moduli $m+1/2=2.08\pm 0.22, 2.77\pm 0.29$, and 2.09 ± 0.22 for the heights 3, 5, and 10 cm, respectively. These results are shown in Fig. 6 with their error bars. The values are very sensitive to the number of data points used, leaving out two or three for any (experimentally always justifiable) reason changes the m values by 0.5. We thus conclude that for all three volumes $\sigma_0 = 1120\pm 50$ Pa and $m = 1.8\pm 0.3$, the Weibull modulus coinciding with Perla's data within the error. From the cumulative curves in Fig. 5 it is obvious that the horizontal shift with height (and volume) is negligible. It is far below the one predicted by Eq. (12) for the dependence on t, which is $\sigma_0(h=3)/\sigma_0(h=10) = (10/3)^{3/(2m+1)} = (10/3)^{3/4} \simeq 2.5$. Unlike predicted by Weibull, there is no shape effect in the probability of failure.

VI. SIZE DEPENDENCE OF RELIABILITY

In order to check any variation of reliability with size, we tested 168 cantilever beams of snow of 222 ± 43 kg/m^3 density, all of quadratic cross section. Fifty-nine of these had a profile $h=t=10.1$ cm, 60 a profile $h=t=5$ cm, and 49 only $h=t=2.5$ cm. Tests were conducted at -5 ± 2 °C. The cumulative failure probabilities as a function of stress are shown in Fig. 7, in linear and logarithmic coordinates, respectively. Evaluation of these curves according to the maximum likelihood method gives values of $\sigma_0 = 5221\pm 205$ Pa and $m+1/2=3.33\pm 0.35$ for the largest, $\sigma_0=5288\pm 300$ Pa and $m+1/2=2.27\pm 0.24$ for the medium, and $\sigma_0=3673 \pm 205$ Pa and $m+1/2=2.55\pm 0.30$ for the smallest profile. These results are shown in Fig. 8. These values are very sensitive to the data used, for example, removing the three strongest specimens of the largest cross section lowers σ_0 to 4880 Pa and increases m to 2.75. According to the weakest link hypothesis, the largest cross section should have the lowest scale parameter σ_0 and the smallest cross section the highest: the opposite is observed. According to Eq. (12), σ_0 should scale for the extremal cross-sectional dimensions tested in our work with the ratio $(10/2.5)^{(5/2)/(m+1/2)}$

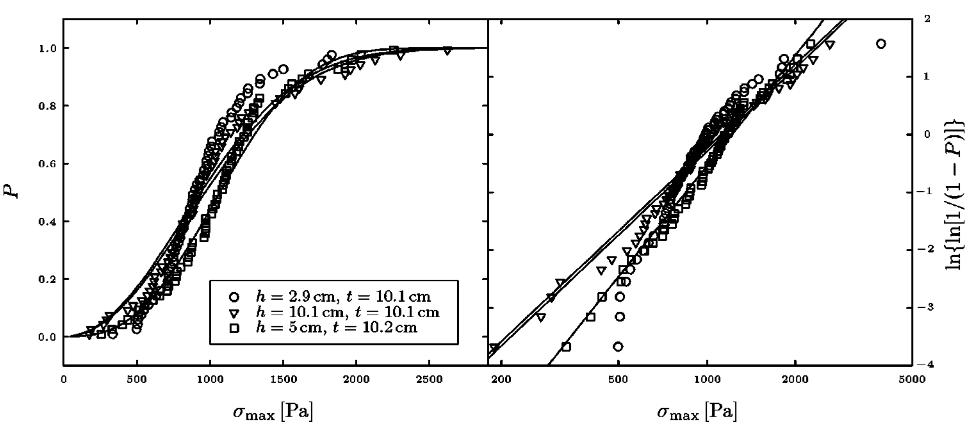

FIG. 5. Weibull analysis of 180 cantilever beams of 107 ± 10 kg/m^3 density, all of width $t=10$ cm, the batches with height $h=3$, 5, and 10 cm consisting of 60 specimens each. Neither the 50% failure strength nor the Weibull modulus m vary appreciably with the shape of the specimens.

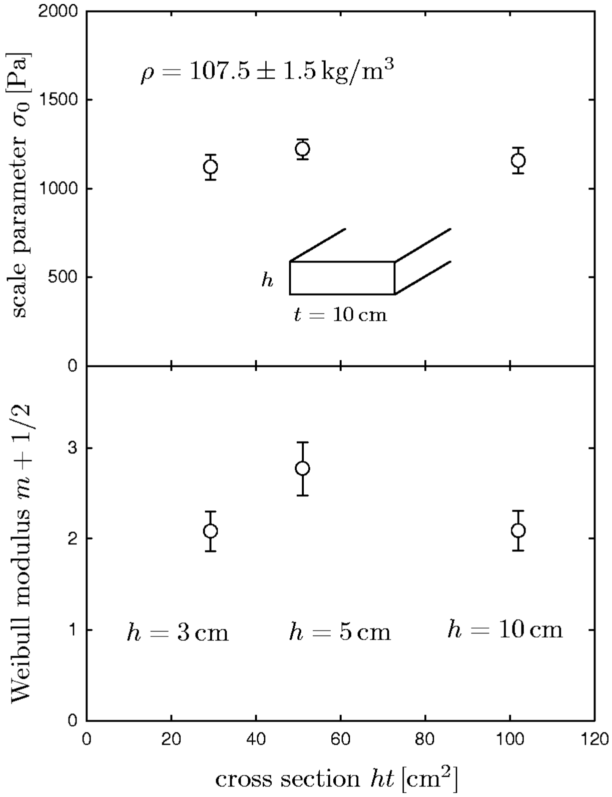

FIG. 6. Scale parameters σ_0 and Weibull moduli m for three beam heights $h=3$,5, and 10 cm, all of width $t=10$ cm.

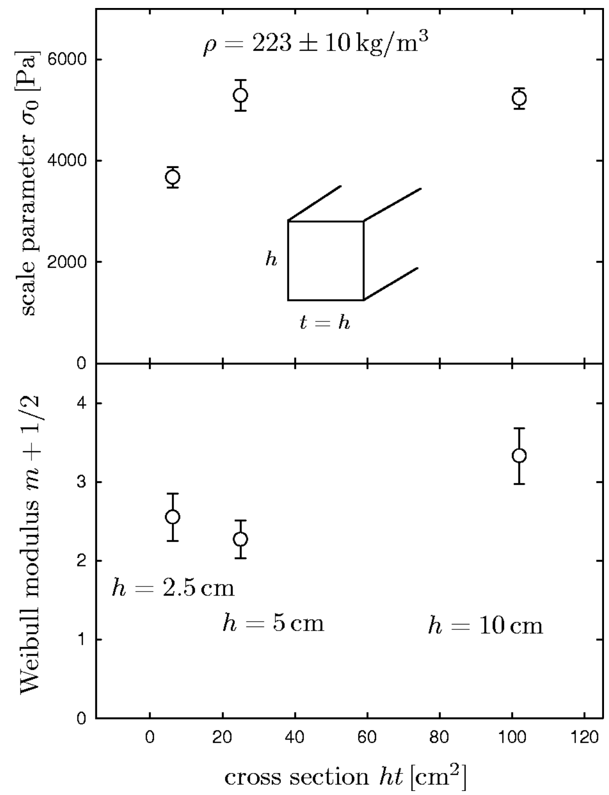

FIG. 8. Scale parameters σ_0 and Weibull moduli m for three beam cross sections.

$=(10/2.5)^{5/4} \simeq 5.7$. This is not observed. Unlike predicted by Weibull, there is no size effect in the probability of failure.

VII. SUMMARY OF EXPERIMENTS

From test on about 600 specimens on snow we conclude the following.

(1) Strength of snow increases with increasing density, from a yield stress of about 400 Pa for 70 kg/m^3 to 10 000 Pa for 250 kg/m^3. The Weibull modulus remains constant at $m = 1.5 \pm 0.5$ for that range of density.

(2) This density dependence is in agreement with structural models of snow that identify it as a foam of ice.

(3) The experimental data confirm that snow density is the controlling parameter for mechanical properties. Of course snows of different microstructure, but of the same density,

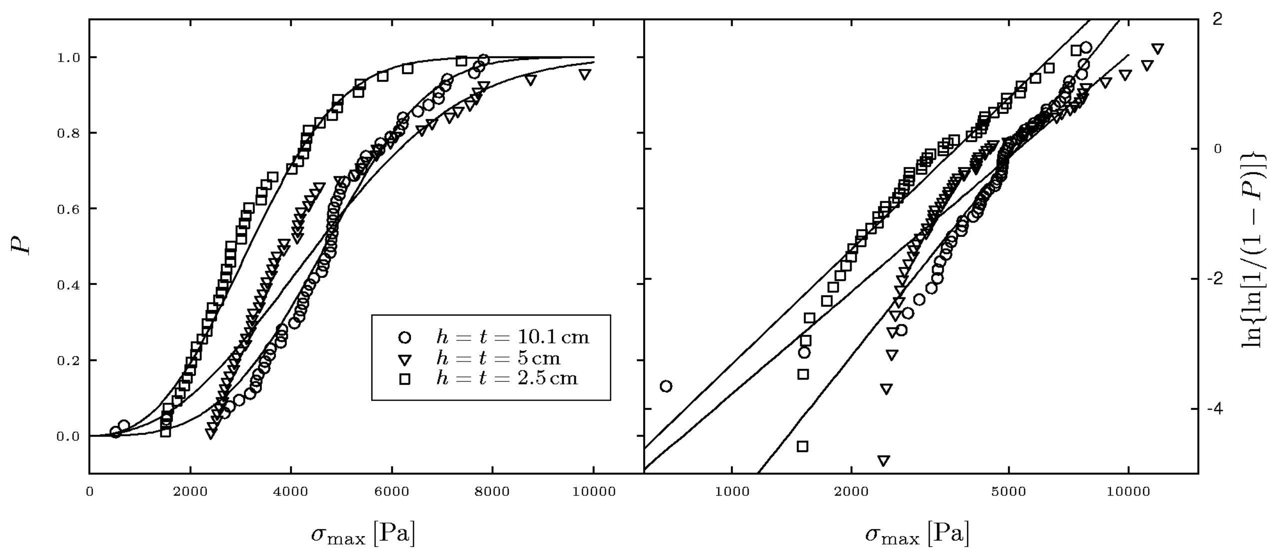

FIG. 7. Weibull analysis of 168 cantilever beams of quadratic cross sections, about a third with $h=t=10.1$,5, and 2.5 cm, respectively. Snow density was 222 ± 43 kg/m^3. Neither the 50% failure strength nor the Weibull modulus m varies significantly with specimen size.

might behave differently, but these structural effects seem to be overridden by density.

(4) Unlike predicted by the weakest link hypothesis, there is neither a size nor a shape dependence of the strength of snow samples in the statistical sense.

(5) The weakest link model underlying the Weibull assumption is not confirmed. The Weibull distribution can only be accepted phenomenologically; it merely describes a two-parameter strength distribution, the mean of which increases and its relative width decreases with increasing density.

(6) The size and shape independence of the mechanical behavior allows the use of experimental data acquired on small, laboratory size specimens for large size snow slabs.

VIII. CONSEQUENCES FOR AVALANCHE RISK

The extraordinarily low values of the Weibull moduli for powder snow ($m=1.5\pm 0.5$ for 70–250 kg/m^3) explain the haphazard and truly random nature of powder snow avalanches. With $m=1.5$ and $\sigma_0=300$ Pa for 70 kg/m^3 there is a 50% probability of failure for a stress $\sigma_{max}=235$ Pa, a 10% probability at $\sigma_{max}=67$ Pa, and still a 1% probability at $\sigma_{max}=14$ Pa. This means that even for very low stresses indeed there is an appreciably finite failure probability that the snow cover fails. As has been known to mountain guides for a long time, freshly fallen powder snow is very unpredictable. Avalanches can go off spontaneously anywhere. For snow of higher density, still unreliable with $m=1.5$, the situation gets better, because under its own weight the stresses in the snow cover increase proportional to the density, while the resisting strength (the scale parameter σ_0) increases quadratically.

The distribution of defects present in our samples, and responsible for the low value of the Weibull modulus, is necessarily smaller than the size of our largest specimens, well below 1 m. Defects and inhomogeneities larger than that, for example, crevasses, cracks, and density fluctuations, are encountered in the field. Such defects provide stress concentrations and are relevant for avalanches, which occur on the scale of 10 m or more. At that scale, we suspect that there is a transition to a Griffith-type abrupt rupture, predicted for systems with decreasing disorder [32], and the existence of crevasses of a few meters size in the snow cover should be treated with conventional mechanics. Slab avalanches triggered by skiers should fall in the domain of conventional linear elastic fracture mechanics. The stress intensity factor is induced by the weight of the skier and the notch length from the length of the skis. One concludes that conventional mechanics can be applied to these problems, which are posed on a length scale larger than our specimens. The material parameters that enter are those measured by us on the centimeter or meter scale. Our experimental results furnish the input to macroscopic avalanche and fracture mechanics of snow. The size independence of these parameters up to 100 cm found by us is quite encouraging.

IX. DISCUSSION

The absence of size effects has shown the weakest link hypothesis in the Weibull sense to be inapplicable, even if the Weibull modulus is very low. We did not observe a distinct scaling relation of the strength, which could be expected for granular disordered media [32,33]. Two possible explanations are suggested for this surprising behavior.

(1) Our bending tests cover a length scale of about one and a half orders of magnitude, thus a scaling law could be masked by the limited range of the experimentally accessible specimen dimensions.

(2) We are approaching an asymptotic strength, which was proposed for large structural dimensions in comparison to the characteristic length of the microstructure in a multifractal scaling law model [34]. Tests towards small scales would clarify this question, but these experiments would be very challenging.

It would be desirable to measure the temperature dependence of the Weibull modulus and the scale parameter. There is no doubt that snow in Arctic or Antarctic conditions, at $-100\,°$C (two-thirds of the melting temperature), might show significantly different behavior from the one measured by us (at 98% of the melting point). On the other hand, it seems unlikely to say that at, say $-20\,°$C, the values of the Weibull modulus and the strength parameter would fall outside our confidence limits. An investigation of temperature effects would really have to address the behavior at extreme conditions, and not at temperatures that in Europe would justly be called cold.

Because it is notoriously difficult [35] to distinguish between different reliability distributions based on failure statistics alone, we have subjected our data to only a Weibull-type analysis, although other hypotheses, for example, Gumbel's one [36], cannot be excluded. At present the microstructure of snow is characterized by average density only. No information on spatial variation of density, density of defects, presence of microcracks, or distribution of porosity is available to us, nor to snow researchers in general. This is the reason why we judged futile the attempt to distinguish between different failure statistics.

In idealized models it is, in principle, possible to find correlations between microstructure and failure statistics; each microstructure leads to a characteristic reliability distribution, but since the former is largely unknown, and the latter is difficult to identify, we have not attempted to do so.

X. CONCLUSION

Inhomogeneously stressed specimens of snow ($30 < \text{density} < 300$ kg/m^3) break at the point of maximum stress σ_{max}, independent of size and shape, with a probability

$$P(\sigma_{max})=1-\exp\left[-\left(\frac{\sigma_{max}}{\sigma_0}\right)^m\right]. \tag{20}$$

The stress parameter varies nearly quadratically with the density of snow, $\sigma_0\approx\rho^{1.85}$, where ρ is the density in kg/m^3. The exponent $m=1.5\pm 0.5$ is independent of density. Although formally identical to a Weibull distribution, the failure law, Eqs. (11) and (12), as outlined in Sec. II, does not hold. The original idea is that in a homogeneously loaded specimen a distribution of defects is present, and the most dangerous ones control failure. The theory then adjusts for inhomogeneous stresses by defining an effective volume according to Eq. (8). This implies that occasionally inhomogeneously loaded specimens should not fail at the point of maximum stress σ_{max}, but somewhere else, albeit with a smaller probability; and that shape and size of specimens matter. For snow it has been verified that these effects do not exist. Failure occurs where the stress is maximum.

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